Section 12-1 FIRST DERIVATIVE AND GRAPHS

- Increasing and Decreasing Functions
- Local Extrema
- First-Derivative Test
- Applications to Economics

Increasing and Decreasing Functions

Sign charts (Section 10-2) will be used throughout this chapter. You will find it helpful to review the terminology and techniques for constructing sign charts now.

Explore & Discuss **1** Figure 1 shows the graph of $y = f(x)$ and a sign chart for $f'(x)$, where

$$f(x) = x^3 - 3x$$

and

$$f'(x) = 3x^2 - 3 = 3(x + 1)(x - 1)$$

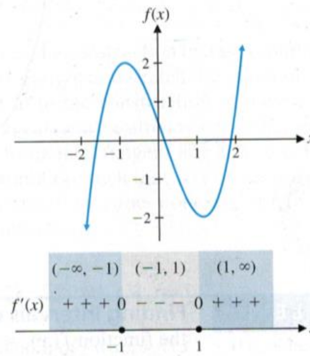

FIGURE 1

Discuss the relationship between the graph of f and the sign of $f'(x)$ over each interval on which $f'(x)$ has a constant sign. Also, describe the behavior of the graph of f at each partition number for f'.

As they are scanned from left to right, graphs of functions generally have rising and falling sections. If you scan the graph of $f(x) = x^3 - 3x$ in Figure 1 from left to right, you will see that

- On the interval $(-\infty, -1)$, the graph of f is rising, $f(x)$ is increasing,* and the slope of the graph is positive $[f'(x) > 0]$.
- On the interval $(-1, 1)$, the graph of f is falling, $f(x)$ is decreasing, and the slope of the graph is negative $[f'(x) < 0]$.
- On the interval $(1, \infty)$, the graph of f is rising, $f(x)$ is increasing, and the slope of the graph is positive $[f'(x) > 0]$.
- At $x = -1$ and $x = 1$, the slope of the graph is 0 $[f'(x) = 0]$.

FIGURE 2

In general, if $f'(x) > 0$ (is positive) on the interval (a, b) (Fig. 2), then $f(x)$ increases (↗) and the graph of f rises as we move from left to right over the interval; if $f'(x) < 0$ (is negative) on an interval (a, b), then $f(x)$ decreases (↘) and the graph of f falls as we move from left to right over the interval. We summarize these important results in Theorem 1.

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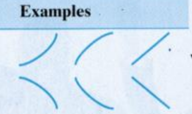
* Formally, we say that the function f is **increasing** on an interval (a, b) if $f(x_2) > f(x_1)$ whenever $a < x_1 < x_2 < b$, and f is **decreasing** on (a, b) if $f(x_2) < f(x_1)$ whenever $a < x_1 < x_2 < b$.

THEOREM 1 INCREASING AND DECREASING FUNCTIONS

For the interval (a, b),

$f'(x)$	$f(x)$	Graph of f	Examples
+	Increases ↗	Rises ↗	
−	Decreases ↘	Falls ↘	



Explore & Discuss 2

The graphs of $f(x) = x^2$ and $g(x) = |x|$ are shown in Figure 3. Both functions change from decreasing to increasing at $x = 0$. Discuss the relationship between the graph of each function at $x = 0$ and the derivative of the function at $x = 0$.

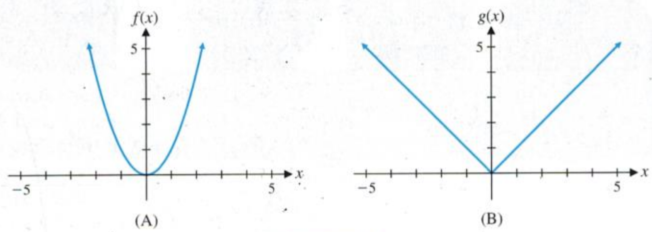


FIGURE 3

EXAMPLE 1

Finding Intervals on Which a Function Is Increasing or Decreasing Given the function $f(x) = 8x - x^2$,

(A) Which values of x correspond to horizontal tangent lines?

(B) For which values of x is $f(x)$ increasing? Decreasing?

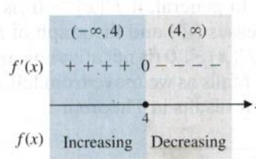
(C) Sketch a graph of f. Add any horizontal tangent lines.

SOLUTION (A) $f'(x) = 8 - 2x = 0$

$$x = 4$$

Thus, a horizontal tangent line exists at $x = 4$ only.

(B) We will construct a sign chart for $f'(x)$ to determine which values of x make $f'(x) > 0$ and which values make $f'(x) < 0$. Recall from Section 10-2 that the partition numbers for a function are the points where the function is 0 or discontinuous. Thus, when constructing a sign chart for $f'(x)$, we must locate all points where $f'(x) = 0$ or $f'(x)$ is discontinuous. From part (A), we know that $f'(x) = 8 - 2x = 0$ at $x = 4$. Since $f'(x) = 8 - 2x$ is a polynomial, it is continuous for all x. Thus, 4 is the only partition number. We construct a sign chart for the intervals $(-\infty, 4)$ and $(4, \infty)$, using test numbers 3 and 5:

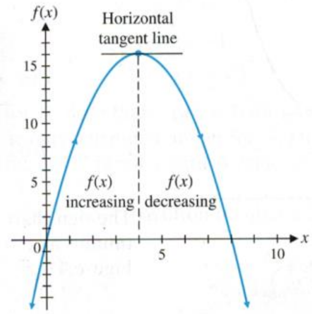


Test Numbers	
x	$f'(x)$
3	2 (+)
5	−2 (−)

Hence, $f(x)$ is increasing on $(-\infty, 4)$ and decreasing on $(4, \infty)$.

(C)

x	$f(x)$
0	0
2	12
4	16
6	12
8	0



MATCHED PROBLEM 1 Repeat Example 1 for $f(x) = x^2 - 6x + 10$.

As Example 1 illustrates, the construction of a sign chart will play an important role in using the derivative to analyze and sketch the graph of a function f. The partition numbers for f' are central to the construction of these sign charts and also to the analysis of the graph of $y = f(x)$. We already know that if $f'(c) = 0$, then the graph of $y = f(x)$ will have a horizontal tangent line at $x = c$. But the partition numbers for f' also include the numbers c such that $f'(c)$ does not exist.* There are two possibilities at this type of number: $f(c)$ does not exist; or $f(c)$ exists, but the slope of the tangent line at $x = c$ is undefined.

DEFINITION Critical Values

The values of x in the domain of f where $f'(x) = 0$ or where $f'(x)$ does not exist are called the **critical values** of f.

INSIGHT

The critical values of f are always in the domain of f and are also partition numbers for f', but f' may have partition numbers that are not critical values.

If f is a polynomial, then both the partition numbers for f' and the critical values of f are the solutions of $f'(x) = 0$.

We will illustrate the process for locating critical values with examples.

EXAMPLE 2 **Partition Numbers and Critical Values** Find the critical values of f, the intervals on which f is increasing, and those on which f is decreasing, for $f(x) = 1 + x^3$.

SOLUTION Begin by finding the partition number for $f'(x)$:

$$f'(x) = 3x^2 = 0, \quad \text{only at } x = 0$$

The partition number 0 is in the domain of f, so 0 is the only critical value of f.

* We are assuming that $f'(c)$ does not exist at any point of discontinuity of f'. There do exist functions f such that f' is discontinuous at $x = c$, yet $f'(c)$ exists. However, we do not consider such functions in this book.

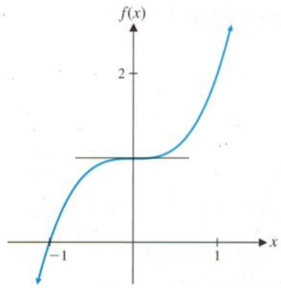
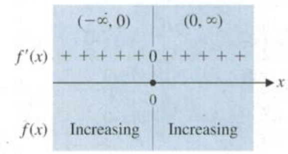


FIGURE 4

The sign chart for $f'(x) = 3x^2$ (partition number is 0) is



Test Numbers	
x	$f'(x)$
-1	3 (+)
1	3 (+)

The sign chart indicates that $f(x)$ is increasing on $(-\infty, 0)$ and $(0, \infty)$. Since f is continuous at $x = 0$, it follows that $f(x)$ is increasing for all x. The graph of f is shown in Figure 4.

MATCHED PROBLEM 2 Find the critical values of f, the intervals on which f is increasing, and those on which f is decreasing, for $f(x) = 1 - x^3$.

EXAMPLE 3

Partition Numbers and Critical Values Find the critical values of f, the intervals on which f is increasing, and those on which f is decreasing, for $f(x) = (1 - x)^{1/3}$.

SOLUTION

$$f'(x) = -\frac{1}{3}(1 - x)^{-2/3} = \frac{-1}{3(1 - x)^{2/3}}$$

To find partition numbers for f', we note that f' is continuous for all x, except for values of x for which the denominator is 0; that is, $f'(1)$ does not exist and f' is discontinuous at $x = 1$. Since the numerator is the constant -1, $f'(x) \neq 0$ for any value of x. Thus, $x = 1$ is the only partition number for f'. Since 1 is in the domain of f, $x = 1$ is also the only critical value of f. When constructing the sign chart for f' we use the abbreviation ND to note the fact that $f'(x)$ is *not defined* at $x = 1$.

The sign chart for $f'(x) = -1/[3(1 - x)^{2/3}]$ (partition number is 1) is

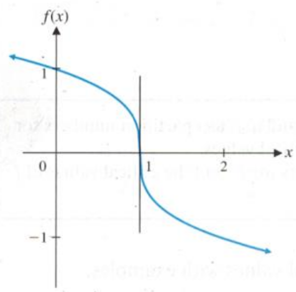
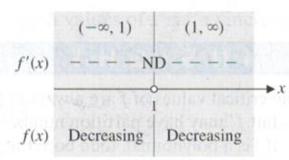


FIGURE 5



Test Numbers	
x	$f'(x)$
0	$-\frac{1}{3}$ (−)
2	$-\frac{1}{3}$ (−)

The sign chart indicates that f is decreasing on $(-\infty, 1)$ and $(1, \infty)$. Since f is continuous at $x = 1$, it follows that $f(x)$ is decreasing for all x. Thus, **a continuous function can be decreasing (or increasing) on an interval containing values of x where $f'(x)$ does not exist.** The graph of f is shown in Figure 5. Notice that the undefined derivative at $x = 1$ results in a vertical tangent line at $x = 1$. In general, **a vertical tangent will occur at $x = c$ if f is continuous at $x = c$ and if $|f'(x)|$ becomes larger and larger as x approaches c.**

MATCHED PROBLEM 3 Find the critical values of f, the intervals on which f is increasing, and those on which f is decreasing, for $f(x) = (1 + x)^{1/3}$.

EXAMPLE 4

Partition Numbers and Critical Values Find the critical values of f, the intervals on which f is increasing, and those on which f is decreasing, for $f(x) = \frac{1}{x - 2}$.

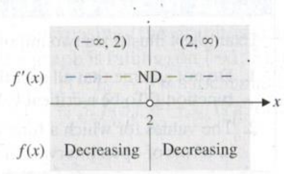
SOLUTION

$$f(x) = \frac{1}{x-2} = (x-2)^{-1}$$

$$f'(x) = -(x-2)^{-2} = \frac{-1}{(x-2)^2}$$

To find the partition numbers for f', note that $f'(x) \neq 0$ for any x and f' is not defined at $x = 2$. Thus, $x = 2$ is the only partition number for f'. However, $x = 2$ is *not* in the domain of f. Consequently, $x = 2$ is not a critical value of f. This function has no critical values.

The sign chart for $f'(x) = -1/(x-2)^2$ (partition number is 2) is



Test Numbers	
x	$f'(x)$
1	−1 (−)
3	−1 (−)

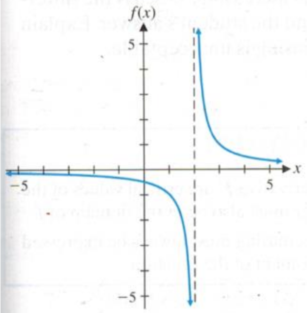


FIGURE 6

Thus, f is decreasing on $(-\infty, 2)$ and $(2, \infty)$. See the graph of f in Figure 6.

MATCHED PROBLEM 4 Find the critical values for f, the intervals on which f is increasing, and those on which f is decreasing, for $f(x) = \frac{1}{x}$.

EXAMPLE 5 Partition Numbers and Critical Values

Find the critical values of f, the intervals on which f is increasing, and those on which f is decreasing, for $f(x) = 8 \ln x - x^2$.

SOLUTION The natural logarithm function $\ln x$ is defined on $(0, \infty)$, or $x > 0$, so $f(x)$ is defined only for $x > 0$. We have

$$f(x) = 8 \ln x - x^2, x > 0$$

$$f'(x) = \frac{8}{x} - 2x \qquad \text{Find a common denominator.}$$

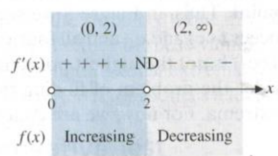
$$= \frac{8}{x} - \frac{2x^2}{x} \qquad \text{Subtract numerators.}$$

$$= \frac{8 - 2x^2}{x} \qquad \text{Factor numerator.}$$

$$= \frac{2(2-x)(2+x)}{x}, \quad x > 0$$

Note that $f'(x) = 0$ at -2 and at 2 and $f'(x)$ is discontinuous at 0. These are the partition numbers for $f'(x)$. Since the domain of f is $(0, \infty)$, 0 and -2 are not critical values. The remaining partition number, 2, is the only critical value for $f(x)$.

The sign chart for $f'(x) = \frac{2(2-x)(2+x)}{x}$, $x > 0$ (partition number is 2), is



Test Numbers	
x	$f'(x)$
1	6 (+)
4	−6 (−)

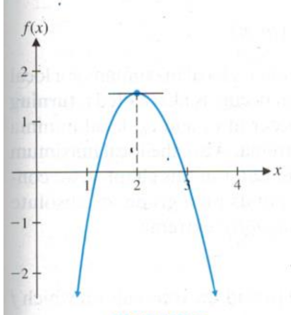


FIGURE 7

Thus, f is increasing on $(0, 2)$ and decreasing on $(2, \infty)$. See the graph of f in Figure 7.

MATCHED PROBLEM 5 Find the critical values of f, the intervals on which f is increasing, and those on which f is decreasing, for $f(x) = 5 \ln x - x$.

Explore & Discuss 3

A student examined the sign chart in Example 4 and concluded that $f(x) = 1/(x - 2)$ is decreasing for all x except $x = 2$. However, $f(1) = -1 < f(3) = 1$, which seems to indicate that f is increasing. Discuss the difference between the correct answer in Example 4 and the student's answer. Explain why the student's description of where f is decreasing is unacceptable.

INSIGHT

Example 4 illustrates two important ideas:

1. Do not assume that all partition numbers for the derivative f' are critical values of the function f. To be a critical value, a partition number must also be in the domain of f.
2. The values for which a function is increasing or decreasing must always be expressed in terms of open intervals that are subsets of the domain of the function.

Local Extrema

When the graph of a continuous function changes from rising to falling, a high point, or *local maximum,* occurs; when the graph changes from falling to rising, a low point, or *local minimum,* occurs. In Figure 8, high points occur at c_3 and c_6, and low points occur at c_2 and c_4. In general, we call $f(c)$ a **local maximum** if there exists an interval (m, n) containing c such that

$$f(x) \le f(c) \qquad \text{for all } x \text{ in } (m, n)$$

Note that this inequality need hold only for values of x near c—hence the use of the term *local.*

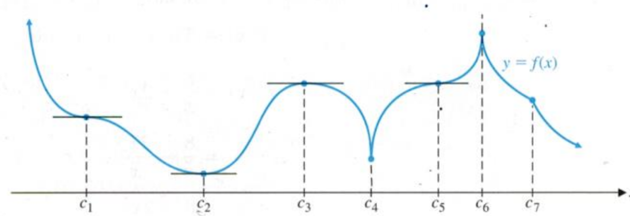


FIGURE 8

The quantity $f(c)$ is called a **local minimum** if there exists an interval (m, n) containing c such that

$$f(x) \ge f(c) \qquad \text{for all } x \text{ in } (m, n)$$

The quantity $f(c)$ is called a **local extremum** if it is either a local maximum or a local minimum. A point on a graph where a local extremum occurs is also called a **turning point.** Thus, in Figure 8 we see that local maxima occur at c_3 and c_6, local minima occur at c_2 and c_4, and all four values produce local extrema. Also, the local maximum $f(c_3)$ is not the highest point on the graph in Figure 8. Later in this chapter, we consider the problem of finding the highest and lowest points on a graph, or absolute extrema. For now, we are concerned only with locating *local* extrema.

EXAMPLE 6 **Analyzing a graph** Use the graph of f in Figure 9 to find the intervals on which f is increasing, those on which f is decreasing, any local maxima, and any local minima.

MATCHED PROBLEM 5 Find the critical values of f, the intervals on which f is increasing, and those on which f is decreasing, for $f(x) = 5 \ln x - x$.

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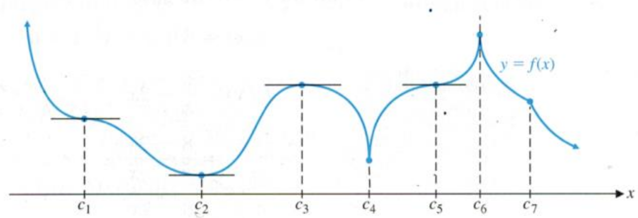

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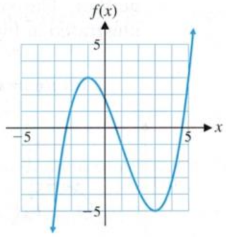


FIGURE 9

SOLUTION The function f is increasing (the graph is rising) on $(-\infty, -1)$ and on $(3, \infty)$ and is decreasing (the graph is falling) on $(-1, 3)$. Because the graph changes from rising to falling at $x = -1$, $f(-1) = 3$ is a local maximum. Because the graph changes from falling to rising at $x = 3$, $f(3) = -5$ is a local minimum. ■

MATCHED PROBLEM 6 Use the graph of g in Figure 10 to find the intervals on which g is increasing, those on which g is decreasing, any local maxima, and any local minima.

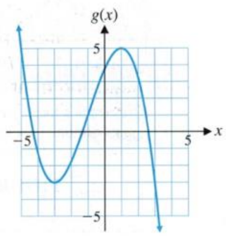


FIGURE 10

How can we locate local maxima and minima if we are given the equation of a function and not its graph? The key is to examine the critical values of the function. The local extrema of the function f in Figure 8 occur either at points where the derivative is 0 (c_2 and c_3) or at points where the derivative does not exist (c_4 and c_6). In other words, local extrema occur only at critical values of f. Theorem 2 shows that this is true in general.

THEOREM 2 EXISTENCE OF LOCAL EXTREMA

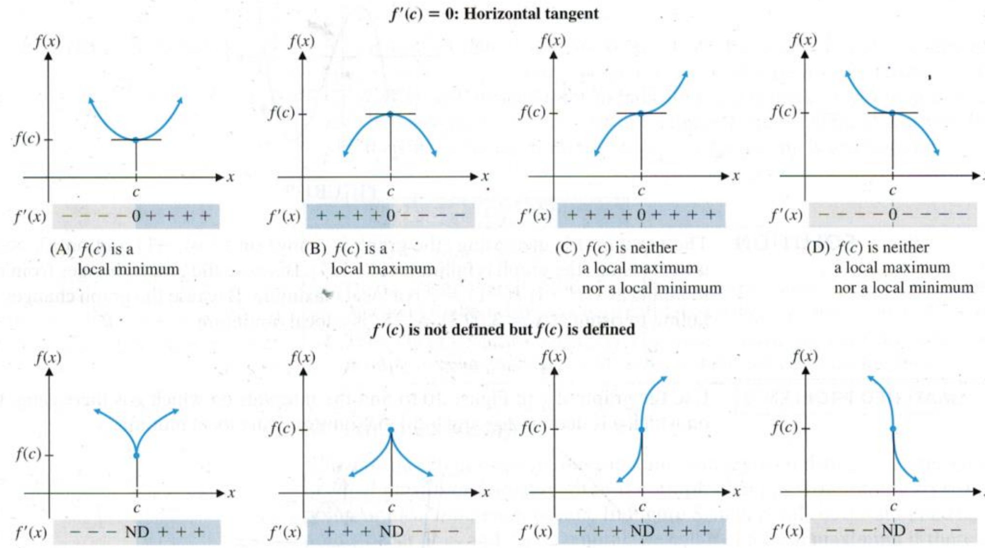
If f is continuous on the interval (a, b), c is a number in (a, b), and $f(c)$ is a local extremum, then either $f'(c) = 0$ or $f'(c)$ does not exist (is not defined).

Theorem 2 states that a local extremum can occur only at a critical value, but it does not imply that every critical value produces a local extremum. In Figure 8, c_1 and c_5 are critical values (the slope is 0), but the function does not have a local maximum or local minimum at either of these values.

Our strategy for finding local extrema is now clear: We find all critical values of f and test each one to see if it produces a local maximum, a local minimum, or neither.

First-Derivative Test

If $f'(x)$ exists on both sides of a critical value c, the sign of $f'(x)$ can be used to determine whether the point $(c, f(c))$ is a local maximum, a local minimum, or neither. The various possibilities are summarized in the following box and are illustrated in Figure 11:



(A) $f(c)$ is a local minimum (B) $f(c)$ is a local maximum (C) $f(c)$ is neither a local maximum nor a local minimum (D) $f(c)$ is neither a local maximum nor a local minimum

(E) $f(c)$ is a local minimum (F) $f(c)$ is a local maximum (G) $f(c)$ is neither a local maximum nor a local minimum (H) $f(c)$ is neither a local maximum nor a local minimum

FIGURE 11 Local extrema

PROCEDURE **First-Derivative Test for Local Extrema**

Let c be a critical value of f [$f(c)$ is defined and either $f'(c) = 0$ or $f'(c)$ is not defined]. Construct a sign chart for $f'(x)$ close to and on either side of c.

Sign Chart	$f(c)$
$f'(x)$: − − − | + + + ; m c n ; $f(x)$: Decreasing | Increasing	$f(c)$ is local minimum. If $f'(x)$ changes from negative to positive at c, then $f(c)$ is a local minimum.
$f'(x)$: + + + | − − − ; m c n ; $f(x)$: Increasing | Decreasing	$f(c)$ is local maximum. If $f'(x)$ changes from positive to negative at c, then $f(c)$ is a local maximum.
$f'(x)$: − − − | − − − ; m c n ; $f(x)$: Decreasing | Decreasing	$f(c)$ is not a local extremum. If $f'(x)$ does not change sign at c, then $f(c)$ is neither a local maximum nor a local minimum.
$f'(x)$: + + + | + + + ; m c n ; $f(x)$: Increasing | Increasing	$f(c)$ is not a local extremum. If $f'(x)$ does not change sign at c, then $f(c)$ is neither a local maximum nor a local minimum.

EXAMPLE 7 **Locating Local Extrema** Given $f(x) = x^3 - 6x^2 + 9x + 1$,

(A) Find the critical values of f.
(B) Find the local maxima and minima.
(C) Sketch the graph of f.

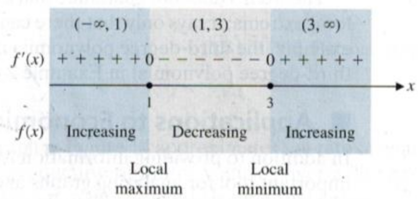
SOLUTION (A) Find all numbers x in the domain of f where $f'(x) = 0$ or $f'(x)$ does not exist.

$$f'(x) = 3x^2 - 12x + 9 = 0$$
$$3(x^2 - 4x + 3) = 0$$
$$3(x - 1)(x - 3) = 0$$
$$x = 1 \quad \text{or} \quad x = 3$$

$f'(x)$ exists for all x; the critical values are $x = 1$ and $x = 3$.

(B) The easiest way to apply the first-derivative test for local maxima and minima is to construct a sign chart for $f'(x)$ for all x. Partition numbers for $f'(x)$ are $x = 1$ and $x = 3$ (which also happen to be critical values of f).

Sign chart for $f'(x) = 3(x - 1)(x - 3)$:



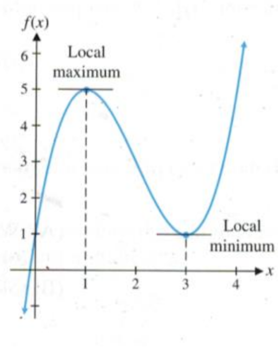
Test Numbers	
x	$f'(x)$
0	9 (+)
2	−3 (−)
4	9 (+)

The sign chart indicates that f increases on $(-\infty, 1)$, has a local maximum at $x = 1$, decreases on $(1, 3)$, has a local minimum at $x = 3$, and increases on $(3, \infty)$. These facts are summarized in the following table:

x	$f'(x)$	$f(x)$	Graph of f
$(-\infty, 1)$	+	Increasing	Rising
$x = 1$	0	Local maximum	Horizontal tangent
$(1, 3)$	−	Decreasing	Falling
$x = 3$	0	Local minimum	Horizontal tangent
$(3, \infty)$	+	Increasing	Rising

(C) We sketch a graph of f, using the information from part (B) and point-by-point plotting.

x	$f(x)$
0	1
1	5
2	3
3	1
4	5



MATCHED PROBLEM 7 Given $f(x) = x^3 - 9x^2 + 24x - 10$,

(A) Find the critical values of f.

(B) Find the local maxima and minima.

(C) Sketch a graph of f.

How can you tell if you have found all the local extrema of a function? In general, this can be a difficult question to answer. However, in the case of a polynomial function, there is an easily determined upper limit on the number of local extrema. Since the local extrema are the x intercepts of the derivative, this limit is a consequence of the number of x intercepts of a polynomial. The relevant information is summarized in the following theorem, which is stated without proof:

THEOREM 3 INTERCEPTS AND LOCAL EXTREMA OF POLYNOMIAL FUNCTIONS

If $f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$, $a_n \neq 0$, is an nth-degree polynomial, then f has at most n x intercepts and at most $n - 1$ local extrema.

Theorem 3 does not guarantee that every nth-degree polynomial has exactly $n - 1$ local extrema; it says only that there can never be more than $n - 1$ local extrema. For example, the third-degree polynomial in Example 7 has two local extrema, while the third-degree polynomial in Example 2 does not have any.

Applications to Economics

In addition to providing information for hand-sketching graphs, the derivative is an important tool for analyzing graphs and discussing the interplay between a function and its rate of change. The next two examples illustrate this process in the context of some applications to economics.

EXAMPLE 8 **Agricultural Exports and Imports** Over the past several decades, the United States has exported more agricultural products than it has imported, maintaining a positive balance of trade in this area. However, the trade balance fluctuated considerably during that period. The graph in Figure 12 approximates the rate of change of the balance of trade over a 15-year period, where $B(t)$ is the balance of trade (in billions of dollars) and t is time (in years).

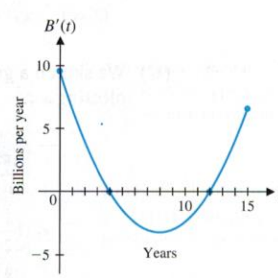


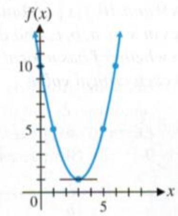
FIGURE 12 Rate of change of the balance of trade

(A) Write a brief verbal description of the graph of $y = B(t)$, including a discussion of any local extrema.

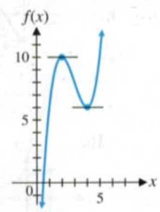
(B) Sketch a possible graph of $y = B(t)$.

Answers to Matched Problems

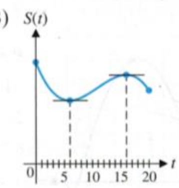
1. (A) Horizontal tangent line at $x = 3$.
(B) Decreasing on $(-\infty, 3)$; increasing on $(3, \infty)$
(C)



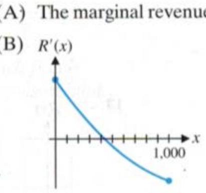
2. Partition number: $x = 0$; critical value: $x = 0$; decreasing for all x
3. Partition number: $x = -1$; critical value: $x = -1$; increasing for all x
4. Partition number: $x = 0$; no critical values; decreasing on $(-\infty, 0)$ and $(0, \infty)$
5. Partition number: $x = 5$; critical value: $x = 5$; increasing on $(0, 5)$; decreasing on $(5, \infty)$
6. Increasing on $(-3, 1)$; decreasing on $(-\infty, -3)$ and $(1, \infty)$; local maximum at $x = 1$; local minimum at $x = -3$
7. (A) Critical values: $x = 2, x = 4$
(B) Local maximum at $x = 2$; local minimum at $x = 4$
(C)



8. (A) The U.S. share of the world market decreases for 6 years to a local minimum, increases for the next 10 years to a local maximum, and then decreases for the final 4 years.
(B)

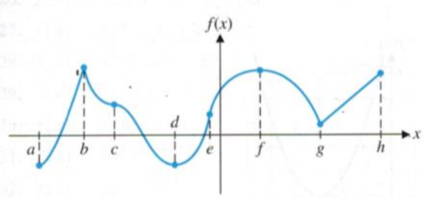


9. (A) The marginal revenue is positive on $(0, 450)$, 0 at $x = 450$, and negative on $(450, 1{,}000)$.
(B)



Exercise 12-1

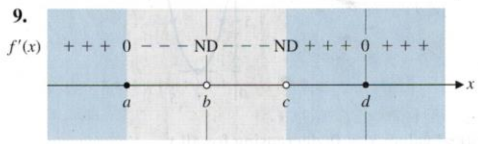
A *Problems 1–8 refer to the following graph of $y = f(x)$:*



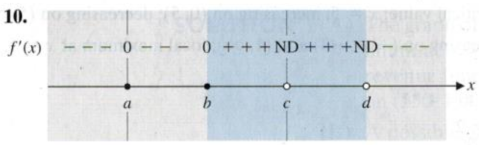
1. Identify the intervals on which $f(x)$ is increasing.
2. Identify the intervals on which $f(x)$ is decreasing.
3. Identify the intervals on which $f'(x) < 0$.
4. Identify the intervals on which $f'(x) > 0$.
5. Identify the x coordinates of the points where $f'(x) = 0$.
6. Identify the x coordinates of the points where $f'(x)$ does not exist.
7. Identify the x coordinates of the points where $f(x)$ has a local maximum.
8. Identify the x coordinates of the points where $f(x)$ has a local minimum.

In Problems 9 and 10, $f(x)$ is continuous on $(-\infty, \infty)$ and has critical values at $x = a, b, c$, and d. Use the sign chart for $f'(x)$ to determine whether f has a local maximum, a local minimum, or neither at each critical value.

9.

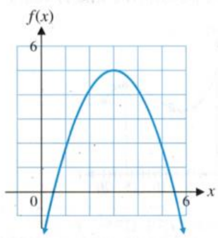


10.

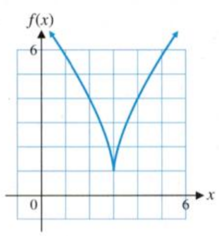


In Problems 11–18, match the graph of f with one of the sign charts a–h in the figure.

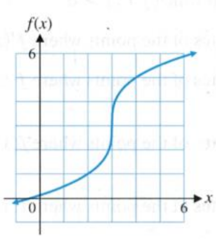
11.



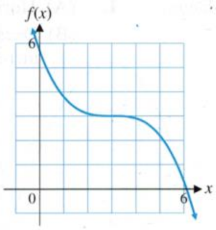
12.



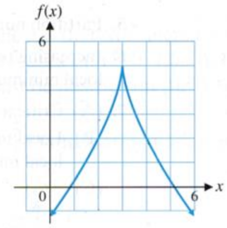
13.



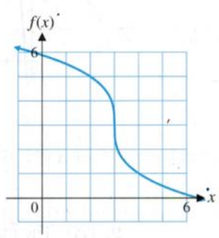
14.



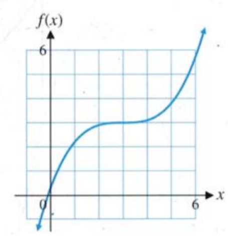
15.



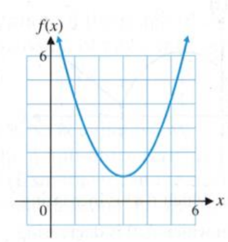
16.



17.



18.



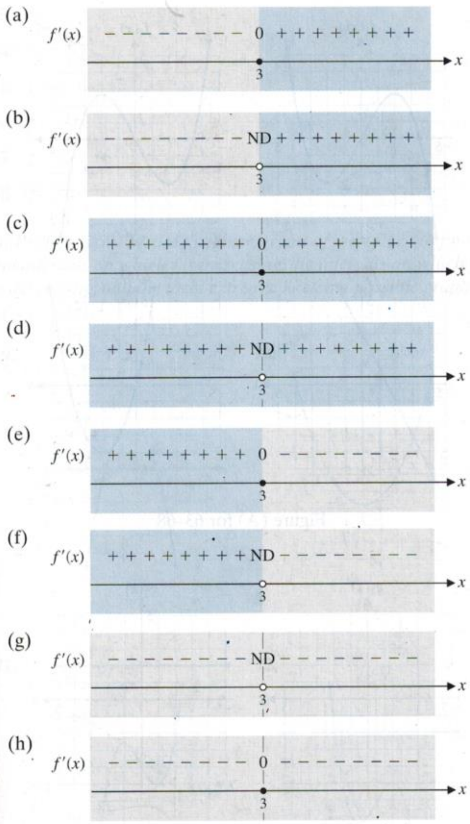


Figure for 11–18

B *In Problems 19–36, find the intervals on which $f(x)$ is increasing, the intervals on which $f(x)$ is decreasing, and the local extrema.*

19. $f(x) = 2x^2 - 4x$

20. $f(x) = -3x^2 - 12x$

21. $f(x) = -2x^2 - 16x - 25$

22. $f(x) = -3x^2 + 12x - 5$

23. $f(x) = x^3 + 4x - 5$

24. $f(x) = -x^3 - 4x + 8$

25. $f(x) = 2x^3 - 3x^2 - 36x$

26. $f(x) = -2x^3 + 3x^2 + 120x$

27. $f(x) = 3x^4 - 4x^3 + 5$

28. $f(x) = x^4 + 2x^3 + 5$

29. $f(x) = (x - 1)e^{-x}$

30. $f(x) = x \ln x - x$

31. $f(x) = 4x^{1/3} - x^{2/3}$

32. $f(x) = (x^2 - 9)^{2/3}$

33. $f(x) = 2x - x \ln x$

34. $f(x) = (x + 2)e^x$

35. $f(x) = (x^2 - 3x - 4)^{4/3}$

36. $f(x) = x^{4/3} - 7x^{1/3}$

In Problems 37–46, use a graphing calculator to approximate the critical values of $f(x)$ to two decimal places. Find the intervals on which $f(x)$ is increasing, the intervals on which $f(x)$ is decreasing, and the local extrema.

37. $f(x) = x^4 + x^2 + x$

38. $f(x) = x^4 + x^2 - 9x$

39. $f(x) = x^4 - 4x^3 + 9x$

40. $f(x) = x^4 + 5x^3 - 15x$

41. $f(x) = x \ln x - (x - 2)^3$

42. $f(x) = 3x - x^{1/3} - x^{4/3}$

43. $f(x) = e^x - 2x^2$

44. $f(x) = e^{-x} - 3x^2$

45. $f(x) = x^{1/3} + x^{4/3} - 2x$

46. $f(x) = \dfrac{\ln x}{x} - 5x + x^2$

In Problems 47–54, find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing. Then sketch the graph. Add horizontal tangent lines.

47. $f(x) = 4 + 8x - x^2$

48. $f(x) = 2x^2 - 8x + 9$

49. $f(x) = x^3 - 3x + 1$

50. $f(x) = x^3 - 12x + 2$

51. $f(x) = 10 - 12x + 6x^2 - x^3$

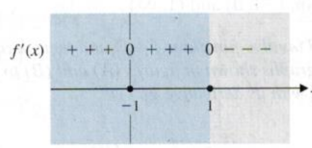
52. $f(x) = x^3 + 3x^2 + 3x$

53. $f(x) = x^4 - 18x^2$

54. $f(x) = -x^4 + 50x^2$

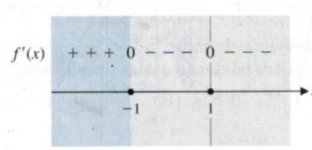
In Problems 55–62, $f(x)$ is continuous on $(-\infty, \infty)$. Use the given information to sketch the graph of f.

55.



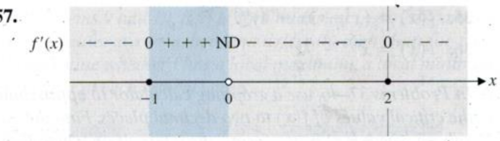
x	−2	−1	0	1	2
$f(x)$	−1	1	2	3	1

56.



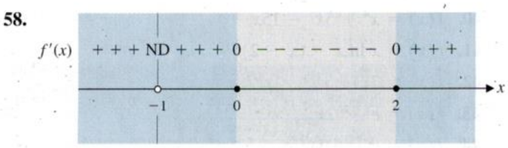
x	−2	−1	0	1	2
$f(x)$	1	3	2	1	−1

57.



x	−2	−1	0	2	4
$f(x)$	2	1	2	1	0

58.



x	−2	−1	0	2	3
$f(x)$	−3	0	2	−1	0

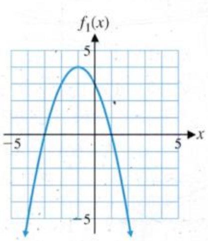
59. $f(-2) = 4, f(0) = 0, f(2) = -4;$
$f'(-2) = 0, f'(0) = 0, f'(2) = 0;$
$f'(x) > 0$ on $(-\infty, -2)$ and $(2, \infty)$;
$f'(x) < 0$ on $(-2, 0)$ and $(0, 2)$

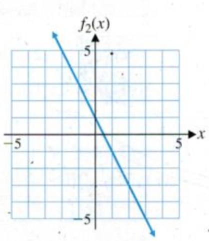
60. $f(-2) = -1, f(0) = 0, f(2) = 1;$
$f'(-2) = 0, f'(2) = 0;$
$f'(x) > 0$ on $(-\infty, -2), (-2, 2)$, and $(2, \infty)$

61. $f(-1) = 2, f(0) = 0, f(1) = -2;$
$f'(-1) = 0, f'(1) = 0, f'(0)$ is not defined;
$f'(x) > 0$ on $(-\infty, -1)$ and $(1, \infty)$;
$f'(x) < 0$ on $(-1, 0)$ and $(0, 1)$

62. $f(-1) = 2, f(0) = 0, f(1) = 2;$
$f'(-1) = 0, f'(1) = 0, f'(0)$ is not defined;
$f'(x) > 0$ on $(-\infty, -1)$ and $(0, 1)$;
$f'(x) < 0$ on $(-1, 0)$ and $(1, \infty)$

Problems 63–68 involve functions f_1–f_6 and their derivatives, g_1–g_6. Use the graphs shown in figures (A) and (B) to match each function f_i with its derivative g_j.





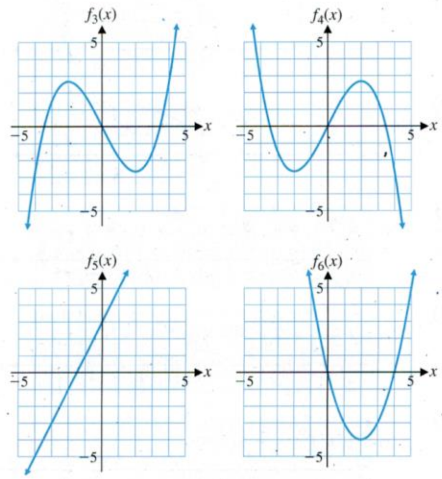


Figure (A) for 63–68

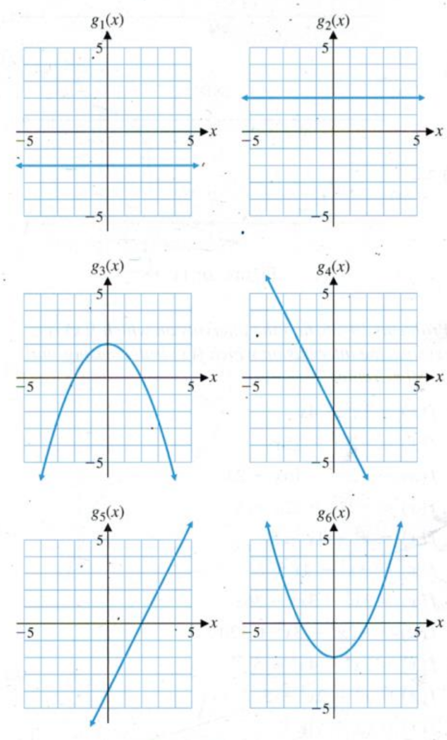


Figure (B) for 63–68

63. f_1

64. f_2

65. f_3

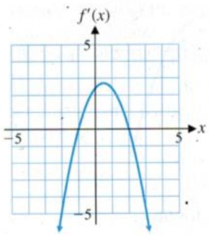
66. f_4

67. f_5

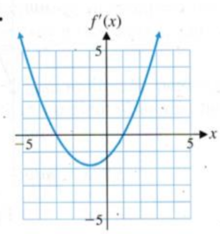
68. f_6

In Problems 69–74, use the given graph of $y = f'(x)$ to find the intervals on which f is increasing, the intervals on which f is decreasing, and the local extrema. Sketch a possible graph of $y = f(x)$.

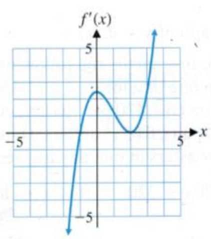
69.



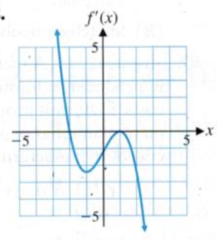
70.



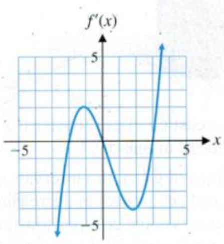
71.



72.



73.



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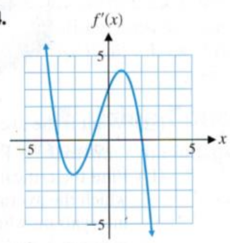
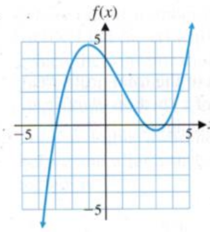
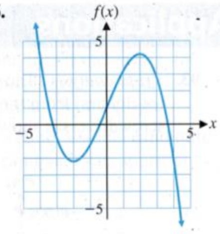


In Problems 75–78, use the given graph of $y = f(x)$ to find the intervals on which $f'(x) > 0$, the intervals on which $f'(x) < 0$, and the values of x for which $f'(x) = 0$. Sketch a possible graph of $y = f'(x)$.

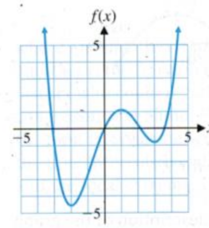
75.



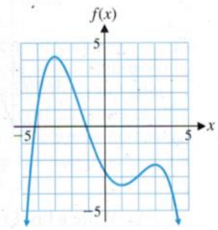
76.



77.



78.



C *In Problems 79–90, find the critical values, the intervals on which $f(x)$ is increasing, the intervals on which $f(x)$ is decreasing, and the local extrema. Do not graph.*

79. $f(x) = x + \frac{4}{x}$

80. $f(x) = \frac{9}{x} + x$

81. $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$

82. $f(x) = 3 - \frac{4}{x} - \frac{2}{x^2}$

83. $f(x) = \frac{x^2}{x - 2}$

84. $f(x) = \frac{x^2}{x + 1}$

85. $f(x) = x^4(x - 6)^2$

86. $f(x) = x^3(x - 5)^2$

87. $f(x) = 3(x - 2)^{2/3} + 4$

88. $f(x) = 6(4 - x)^{2/3} + 4$

89. $f(x) = \frac{2x^2}{x^2 + 1}$

90. $f(x) = \frac{-3x}{x^2 + 4}$

91. Let $f(x) = x^3 + kx$, where k is a constant. Discuss the number of local extrema and the shape of the graph of f if

(A) $k > 0$ (B) $k < 0$ (C) $k = 0$

92. Let $f(x) = x^4 + kx^2$, where k is a constant. Discuss the number of local extrema and the shape of the graph of f if

(A) $k > 0$ (B) $k < 0$ (C) $k = 0$